REDWOOD
INVESTMENT MANAGEMENT

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Skew Portfolio Theory (SPT) utilizes statistical skew to build a dynamic weighted portfolio with potential to capture additional risk premia.

THE THIRD MOMENT IN EQUITY PORTFOLIO THEORY & SPT

Summary

Numbers are nothing more than an abstract way to describe and model natural phenomena. Applied to capital markets, historical numerical data is often utilized as a proxy to describe the behavior of security price movement. Harry Markowitz proposed Modern Portfolio Theory (MPT) in 1952, and since then, it has been widely adopted as the standard for portfolio optimization based on expected returns and market risk. While the principles in MPT are important to portfolio construction, they fail to address real-time investment challenges. In this primer, we explore the efficacy of statistical skew and its application to real-time portfolio management. What we find is that statistical skew can be utilized to build a more dynamic portfolio with potential to capture additional risk premia – Skew Portfolio Theory (SPT). We then apply SPT to propose a new rebalancing process based on statistical skewness (Skew). Skew not only captures risk in terms of standard deviation, but analyzing how skew changes over time, we can measure how statistical trends are changing, thus facilitating active management decisions to accommodate the trends. The positive results led to the creation and management of Redwood's Equity Skew Strategy that is implemented in real-time. In short:

- Skew is a less commonly viewed statistic that captures the mean, standard deviation, and tilt of historical returns in one number.
- A process based on skew or SPT can be used to enhance portfolio construction in a real-time, forward-looking, implementable way, while addressing many problems of MPT.
- A portfolio dynamically adapting to changing skew can potentially be overweight to asset classes in opportune times to capture risk premia efficiently.

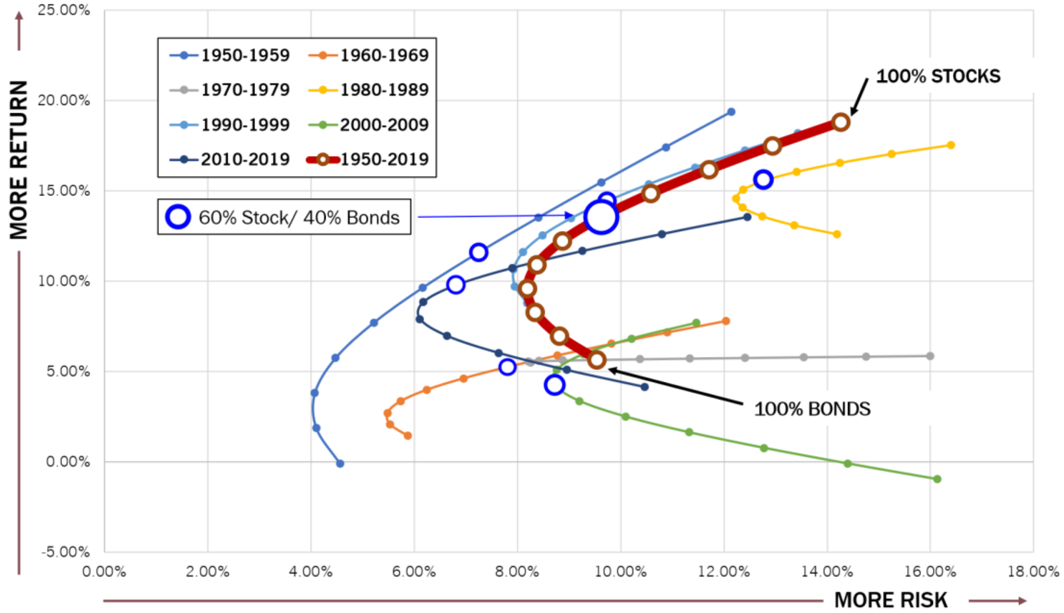
Introduction

Modern Portfolio Theory (MPT) was pioneered by Markowitz in 1952. Surprisingly, almost seven decades later, MPT is still used as the standard for investment portfolio construction. MPT utilizes statistics, such as the average, variance, and correlation of asset returns to build the most "efficient" portfolio defined by the maximum possible expected return for a given level of risk, termed the "efficient frontier." Also known as mean-variance analysis, MPT derives the expected return on an asset utilizing a historical mean and variance (or standard deviation) as historical risk. Portfolio return is therefore the pro-rata weighted combination of the constituent assets' returns, and portfolio risk is a function of the correlations of the assets held with respect to their variance.

A simple example is how MPT led to the widely used "60% Stock, 40% Bond portfolio." Utilizing data since the 1950's, graphing the return of a portfolio of stocks and bonds resulted in a "C-hook" shape, which illustrated that a portfolio of 60% stocks achieved higher returns than a 100% bond portfolio without substantially increasing risk. The clear problem with the efficient frontier, however, is highlighted in Figure 1 below. Because bond and equity returns are NOT consistent with the theorized expected return and variance based on MPT, different time periods will result in widely different "optimal portfolios."

The most "efficient" portfolio calculated by the efficient frontier will change depending on the time frame of the data used. The most efficient portfolio in the last 10 years, is most likely not going to be the most efficient portfolio in the next 10 years.

Figure 1: Commonly Cited "Efficient Frontier" Varies Over Time (5-Year Time Periods)



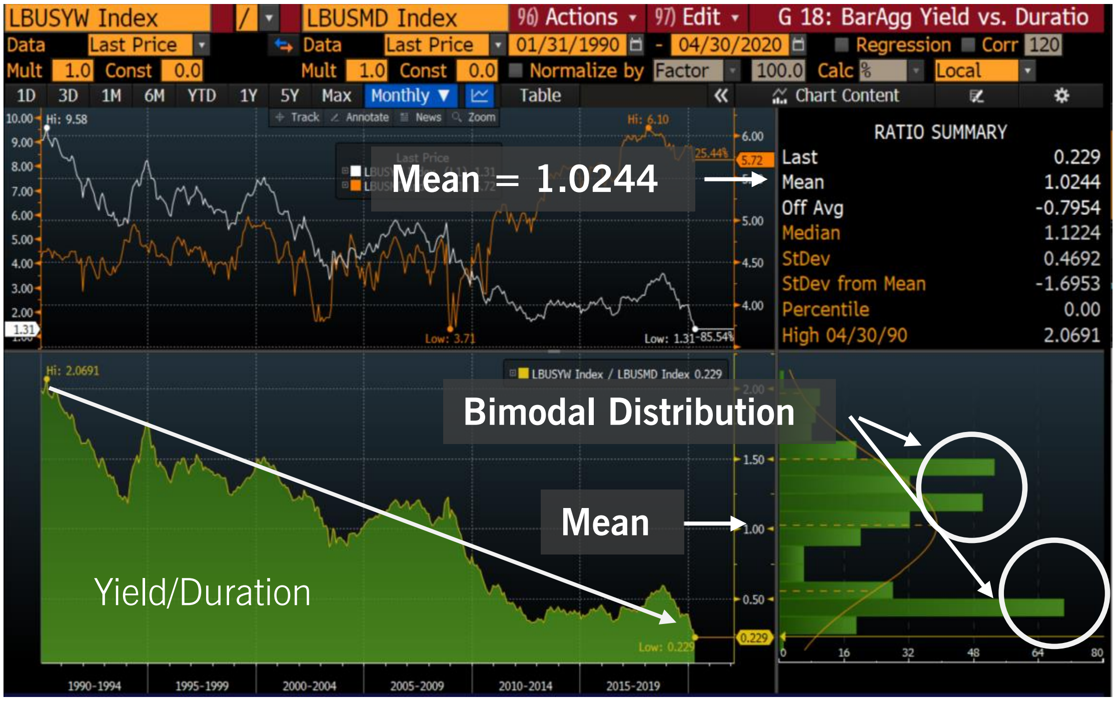
Sources: Morningstar, Bloomberg, Redwood. For illustration purposes only. Stocks is represented by the S&P 500 Index. Bond data utilized is Ibbotson bond data that utilizes U.S. Long-Term Treasury Bonds as a proxy. An investor cannot invest directly in an index.

Certainly, it goes without saying that utilizing past historical performance is not indicative of future results. This is the fundamental problem with utilizing MPT to attempt to build the "most efficient" portfolio for the future. As illustrated above, the most efficient portfolio changes over time and can only be calculated after the fact. **This makes the application of MPT to realize the most efficient portfolio across a forward looking time-frame for investment management highly improbable.** The most efficient portfolio in the last 10 years is most likely not going to be the most efficient portfolio in the next 10 years. The reality is that asset ***returns***, thus ***risk*** and ***correlation, change over time.***

MPT also describes asset returns with minimum detail and information. MPT utilizes statistics to describe asset behavior relevant to investors, such as return and risk based on the historical distribution of returns over time. More specifically, "moments", quantitative measures of a shape of a function, are utilized to measure return and risk. The first moment, an average, is utilized as expected value, and the second moment, variance, is utilized as risk of a security. Like most theories, MPT is built on many assumptions, one being that asset returns are normally distributed in a perfect bell curve. This allows information to be lost and the math to be simplified to the two moments of expected return and variance. Real asset returns however, are far from normally distributed and the distribution changes over time. For example, in Figure 2, the Barclays Aggregate's return per unit of risk has changed so much post 08, that pre-08 and post-08 has a bimodal distribution, meaning having two peaks instead of one defined by a normal distribution. If MPT assumptions were used at the beginning of the full time period, the average would be far from realistic expectations given what actually occurred.

A bimodal distribution has two separate distinct peaks. A simple mean does not represent any useful information on where the aggregated data lies.

Figure 2: Return Per Risk (Duration) of Barclays Aggregate Changed Dramatically



Sources: Bloomberg, Redwood. For illustration purposes only. An investor cannot invest directly in an index

To increase the amount of information from a set of historical distributions, we utilize the third moment of a probability distribution curve: skew, $\bar{\mu}_3$. Different from MPT, our Skew Portfolio Theory (SPT) assumes:

1. Historical asset returns are not distributed in a normal curve, but have a skew $\bar{\mu}_3 \neq 0$
2. Incremental asset return data will change the skew over time
3. The skew of asset returns converge towards 0 as asset behavior normalizes in rolling time frames such that $\lim_{n\to\infty} \bar{\mu}_3\{x_1, x_2, x_3 \ldots x_n\} = 0$

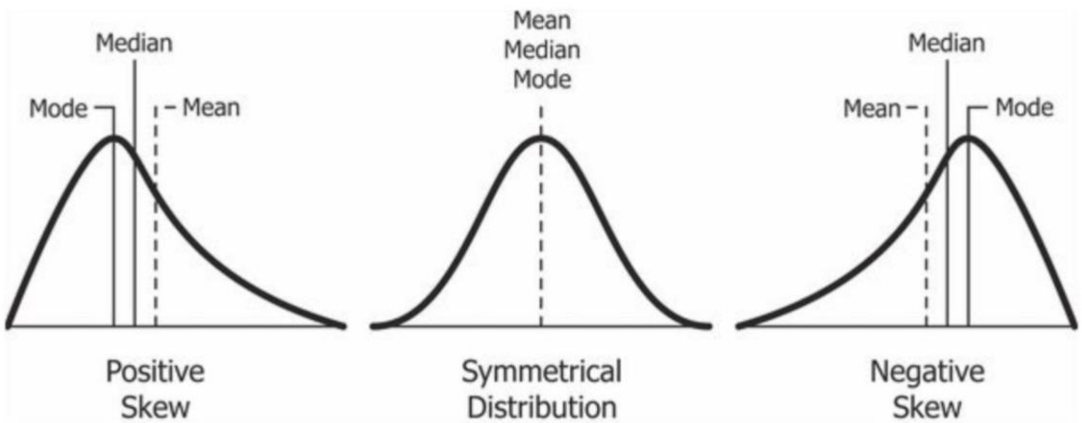
Assumptions for SPT may be more realistic in the implementation of a real-time investment management process.

The assumptions above in SPT may be more realistic in the implementation of a real-time investment management process. First (more fact than assumption), historical asset returns can be empirically shown to not be normally distributed (for the purpose of this study, we aren't including risk-less assets). Because historical returns are not normally distributed, they exhibit some skew either positively or negatively. However, over time, asset returns converge towards a normal distribution or a skew of 0. This is because as environments change and asset returns respond, the incremental data will eventually create a new center of mass. For example, Figure 2 above shows that incrementally, data for the Barclays Aggregate Bond Index eventually accumulated at a new average after 2008. This therefore can create an opportunity to respond to implied asset return direction as its distribution moves toward a normal curve.

Advancing To The Third Moment - Skewness

Skew measures the amount of asymmetry of a distribution around its mean. This measures the amount that a distribution leans in one direction or another around its average value. In a positive skew, the mean of the distribution is shifted to the right relative to its median and mode due to a right-side tail. The opposite is true with a negative skew.

Figure 3: Illustrating Skewness Of A Distribution



Sources: Wikipedia. For illustration purposes only.

Mathematically, SPT analyzes historical performance in the same way as MPT for the first two moments and has the standardized notation such that for continuous variable x, the m^{th} moment $\bar{\mu}_m$:

$$\bar{\mu}_m = \int_{-\infty}^{\infty} (x-c)^m f(x)dx$$

- The first moment, mean μ of a set of historical returns or Expected Return where m = 1 and a historical data set X, is typically denoted as:

$$\mu \equiv \mathrm{E}[\mathrm{X}]$$

Because the calculation of skew utilizes the 3rd power different from standard deviation which utilizes the 2nd, skew can be negative or positive, which captures information about direction in addition to magnitude.

- The second moment, variance σ of a set of historical returns commonly used as "risk" where m = 2 and a historical data set of X, is denoted as:

$$\sigma \equiv \sqrt{\mathrm{E}[(x-\mu)^2]}$$

- The third moment, s we approach a normalized 3rd moment m = 3, we can utilize a normalized central moment $\bar{\mu}_m$ above to derive the m^{th} central moment of X as $\frac{\mathrm{E}[(X-\mu)^m]}{\sigma^m}$ to arrive at the skewness of n data points as expressed as:

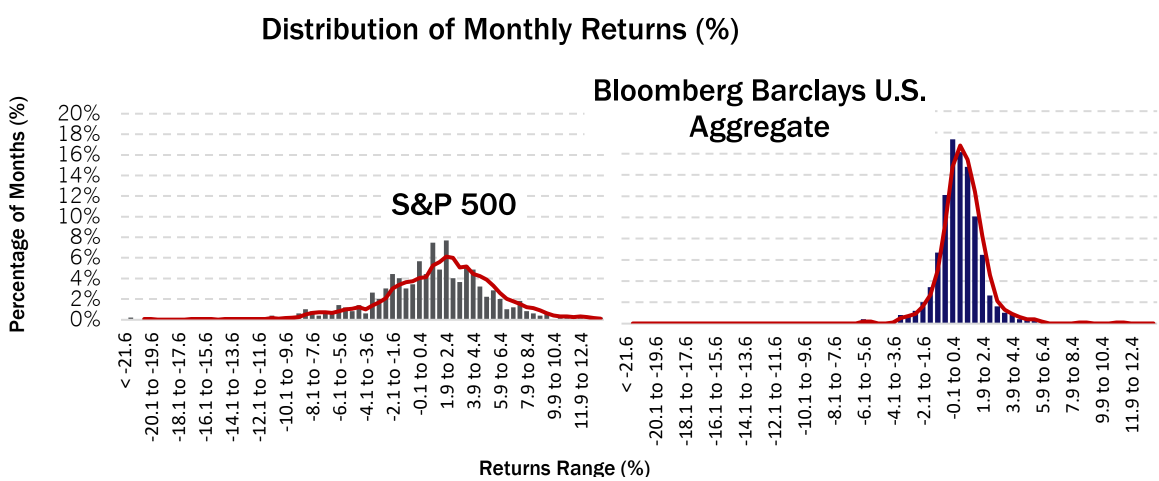
$$\bar{\mu}_3 = \gamma = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{(X-\mu)}{\sigma} \right)^3$$

- Because the differences between the mean and the standard deviation are cubed, skewness effectively measures a larger degree of deviation from the mean versus variance or standard deviation, which helps measure the potential of tail risk or larger deviations away from the mean. In addition, a 3rd power (versus 2nd) allows skewness to mathematically be a positive or negative, indicating the direction of skew (variance is always positive), which gives more information about the tilt of the distribution.

Consider the below calculations of skew of two popular U.S. indices, the Bloomberg Barclays Aggregate Bond Index, and the S&P 500 Index.

Figure 4: Distribution of Historical Returns Display Skew

The actual distributions of historical returns are not normal curves. They tend to skew negatively or positively.



	Mean	Standard Deviation	Skewness
S&P 500	0.75%	4.30%	-0.68
Bloomberg Barclays U.S. Aggregate	0.61%	1.55%	0.77

Sources: Zephyr, Redwood, Bloomberg. Data from July 1995 – March 2020. For illustration purposes only. An investor cannot directly invest in an index.

Empirical Support of SPT: Observations vs. Assumptions

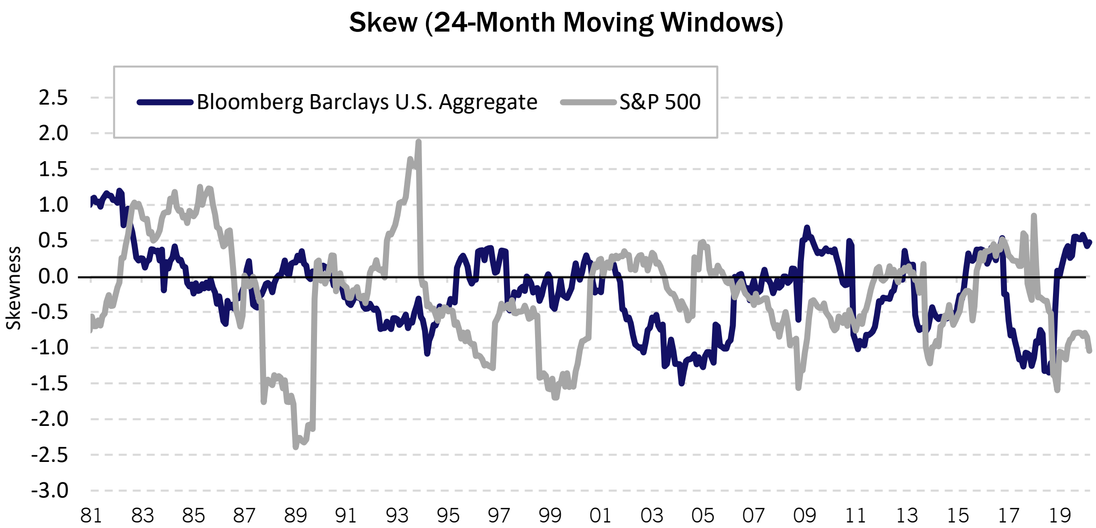
Shown above in Figure 4, the skew of asset returns is not (and most likely will never be) consistent at 0, such as a normal distribution. The above data however is a one-time calculation of skew over the period of July 1995-March 2020. Figure 2 above shows that the time relevancy of distribution data can be critical to properly analyzing historical returns in context. Therefore, it is important to understand how statistics can change over periods of time. For example, in Figure 5, a 2-year (24-month) rolling period data set of different returns shows the skew of asset classes changing over time. The changes aren't captured in a one-time distribution number. In addition, as the data set becomes smaller, in a corollary to assumption #3,

> The skew of asset returns converges toward 0 as asset behavior normalizes in rolling time frames such that $\lim_{n\to\infty} \bar{\mu}_3\{x_1, x_2, x_3 \dots x_n\} = 0$

SPT builds upon underlying cases of empirical data, rather than theoretical assumptions used for MPT – a perfect normal distribution curve never occurred, so why pretend that it has? Statistics such as skew changes over time.

We can expect that not only does the skew change overtime, the direction of the skew also changes with greater frequency over time in shorter time frames. Because a change in direction implies the skew crossed 0, the data also supports, that over time, a convergence of skew towards 0 occurs. This is a mathematical way of describing that a lot of something different, is a new normal. Granted, limitations will always persist when utilizing historical data, SPT builds upon underlying cases of empirical data, rather than theoretical assumptions used for MPT. A perfect normal distribution curve never occurred, so why pretend that it has?

Figure 5: Skew Over Time Of Barclays Aggregate Bond Index & S&P 500 Index



Sources: Zephyr, Redwood. Data from January 1981 – March 2020. For illustration purposes only. An investor cannot directly invest in an index.

Skew Portfolio Theory (SPT) – An Enhanced Portfolio Process

Skewness by itself can be a strong measurement of asset returns because the calculation of skew requires information such as mean and variance.

As shown, skewness by itself is a strong measurement of asset returns. This is because within the calculation of a single skew number, it also utilizes all preceding moments such as mean and variance. The natural next step is to explore how skew can be incorporated into a portfolio of multiple assets with varying skews that change over time. One of the principles of MPT asserts that the risk of a portfolio, or portfolio variance, is defined by the pro rata weight of the risk attributed by the assets $\sum_i w_i\sigma_i$ and their correlation ρ_{ij} . In the widely utilized portfolio variance equation from MPT below, the negative correlation of two assets, $-1 < \rho_{ij} < 1$ can lessen the overall portfolio variance $\sum_i w_i\sigma_i$. The formula can be extended to a greater number of assets utilizing a correlation matrix for all constituents of a portfolio (P).

$$\sigma_P^2 = \sum_i w_i\sigma_i + \sum_i \sum_{j\neq i} w_i w_j\, \sigma_i \sigma_i \rho_{ij}$$

The challenge is that returns are not normally distributed and consistent over time. Because SPT asserts that the skew of an asset changes over time based on changing returns, correlation between two assets change over time as well. The general goal of portfolio construction is to lessen the amount of risk in investing by diversifying between multiple assets.

Consider the popular 60% stock (s) and 40% bond (b) portfolio, the portfolio variance would be the below:

$$\sigma_{s,b}^2 = (0.60_s\sigma_s + 0.40_b\sigma_b) + (0.60 * 0.40 * \ \sigma_s\sigma_b\rho_{sb})$$

Investment managers should actively manage the only controllable variable to moderate risk, that is the weight of the asset class exposure. Position size is everything.

The calculation of the portfolio variance of a fixed weight 60% stock and 40% bond portfolio immediately presents a challenge to the goal of minimizing portfolio variance and the argument of passive index rebalancing. This is because the variables of portfolio risk and correlation change due to market movements, so solving for fixed weights will virtually always yield different risk results for the portfolio. This further reinforces that a truly forward-looking efficient portfolio cannot be achieved utilizing MPT's efficient frontier. Instead, investment managers should actively manage the only controllable variable to moderate risk, or the variance of σ_P^2 ,which is the weight w of the underlying constituent assets. Therefore, to manage real-time portfolio risk, the weights must change dynamically with frequency.

SPT provides a framework to manage and construct a portfolio by quantitatively and dynamically changing the weights utilizing higher power math of skew.

SPT provides a framework to manage and construct a portfolio by quantitatively and dynamically changing the weight w by utilizing the higher power math of skew (a dynamic weight based on variance itself may not work because of the same problem of assuming variance is the same in any time period). Recall that the equation of skew as a function of the mean μ and variance of σ as a denominator. Therefore, portfolio variance σ_P^2 would be:

$$\sigma_P^2 = \sum_i w(f(\gamma_i, \mu_i, \sigma_i))_i\sigma_i + \sum_i \sum_{j \neq i} w(f(\gamma_i, \mu_i, \sigma_i))_i \, w\big(f\big(\gamma_j, \mu_i, \sigma_i\big)\big)_j \, \sigma_i\sigma_i\rho_{ij}$$

This framework utilizes more information dynamically than static weights based on the best historical average return. In addition, note that because the weight is derived based on skew γ, changes in mean μ and variance σ are directly accounted for during changes. Furthermore, the correlation between two assets are also a function of the mean μ and variance σ of the different assets. Finally, because SPT theorizes the convergence of skew toward 0 over time, this provides potential actionable changes to weight a portfolio to its optimal point assuming a **forward-looking convergence** with a high degree of statistical significance to efficiently capture risk premia. For example:

> Suppose a two asset portfolio where the assets are highly correlated and both almost have a normal distribution with similar risk and therefore equally weighted:
>
> Time $= t_n$; $\rho_{1,2} \sim 1$; $\gamma_1 \sim 0$; $\gamma_2 \sim 0$; $\sigma_1 \sim \sigma_2$ $w_1 = w_2$
>
> If asset 2 becomes more volatile and diverges
>
> Time $= t_{n+1}$; $\rho_{1,2} < 1$; $\gamma_1 \sim 0$; $|\gamma_2| > 0$; $\sigma_1 < \sigma_2$
>
> Because $\rho_{1,2} < 1$ and is lower than $\rho_{1,2}$ at time t_n , based on SPT, we can overweight such that $w_2 < w_1$ utilizing the proportion of change in skew while maintaining the portfolio variance of σ_P^2 . Because the risk of asset 2 effectively increased, a dynamic weight based on SPT can capture the risk premia presented, while maintaining the same portfolio risk.

SPT can mathematically capture risk premia, while maintaining the same level of portfolio variance.

A portfolio model based on SPT can therefore theoretically dynamically allocate to “higher” risk assets at times where relative skew changes to improve the forward-looking efficiency of a portfolio, even if the assets are not negatively correlated to each other.

SPT Application

Following SPT, we can weight a multi-asset portfolio dynamically utilizing theory discussed above. Portfolio weights are adjusted overtime as a function of relative skew in a matrix. To test, we utilize five broad equity categories with relatively high correlation: large-cap value (S&P 500 Value Index), large-cap growth (S&P 500 Growth Index), small-cap value (Russell 2000 Value Index), small-cap growth (Russell 2000 Growth Index), and emerging market equity (Dow Jones Emerging Market Index). The portfolio is run versus a non-optimized equal weight blend of the aforementioned equity categories.

Because all the asset classes in this study are equities, their correlations are relatively high.

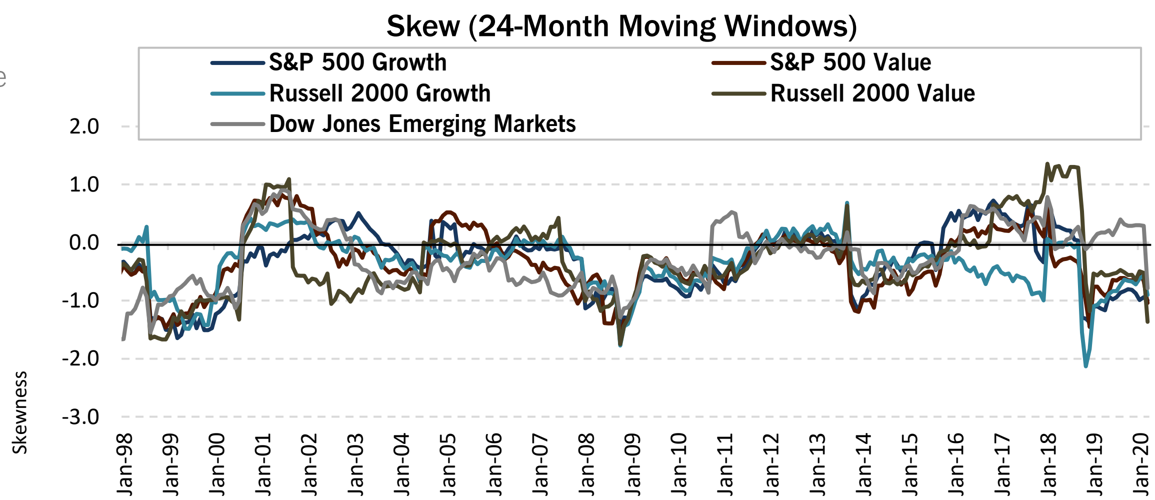
Figure 6: Correlation Matrix

	1) S&P 500 Growth	2) S&P 500 Value	3) Russell 2000 Growth	4) Russell 2000 Value	5) Dow Jones Emerging Markets
1) S&P 500 Growth	1.00				
2) S&P 500 Value	0.84	1.00			
3) Russell 2000 Growth	0.80	0.73	1.00		
4) Russell 2000 Value	0.68	0.86	0.85	1.00	
5) Dow Jones Emerging Markets	0.70	0.70	0.67	0.63	1.00

Sources: Zephyr, Redwood. Data from July 1995 – March 2020. For illustration purposes only. An investor cannot directly invest in an index.

However, their skew also fluctuates over time about 0 at different points in times, providing opportunity to overweight and underweight to potentially capture risk premia.

Figure 7: Skew Over Time



Sources: Zephyr, Redwood. Data from July 1995 – March 2020. For illustration purposes only. An investor cannot directly invest in an index.

A study on dynamic weighting process based on skew demonstrates outperformance over a fixed, equal weight blend.

Figure 8: Hypothetical Results Of Study Based On Dynamic Skew Weights

	Equity Skew Strategy	Equal Weight To Indices
Annualized Return	7.67%	6.92%
Maximum Drawdown	-51.89%	-53.61%
Pain Index	7.39%	8.64%
Standard Deviation	17.00%	17.30%
Sharpe Ratio	0.33	0.28
Downside Deviation	11.82%	12.28%
Sortino Ratio	0.65	0.56
Skewness	-0.71	-0.84
Average Up Return	3.66%	3.57%
Average Down Return	-4.18%	-4.48%

Sources: Zephyr, Redwood. Data from October 1996 – March 2020. For illustration purposes only. An investor cannot directly invest in an index. Hypothetical results utilize index returns and is not representative of any investable investment strategy. See important disclosures at the end for further information.

Conclusion

SPT addresses many of the real time problems with traditional portfolio management, and proposes a viable solution with demonstrated efficacy. We utilize our theory in real time for the Redwood Equity Skew Strategy.

Empirical data and their statistics can only help suggest the expectation of what the returns of an asset may look like in the future. In reality, new environments can shape and change how these asset classes behave. While MPT provides a starting point for portfolio construction, the simplicity of its math may lead to overutilization and reliance when it comes to attempting to build an "optimal" portfolio. Exacerbating the situation, since the 1950's, available technology has advanced tremendously – investors can now push a "button" to "optimize" their portfolio. The flaw, whether MPT, or picking the best historical performing stocks, building yesterday's optimal portfolio may not be the best risk-adjusted portfolio for tomorrow. No portfolio process is without flaws. At the end of the day, SPT still utilizes historical trends. However, SPT addresses many of the big real time problems with portfolio management and proposes a potential solution with demonstrated efficacy.

At Redwood, we focus on a blank canvas approach. While the academics of existing portfolio theory demonstrate important principles for portfolio construction, our approach is to analyze methodologies to effectively implement theory into managing real-time portfolios for client assets (not just academia). Skew Portfolio Theory can have many further implications, setting the stage to utilize deeper math to respond to more complex problems in a changing environment. The efficacy is continually demonstrated and assessed as it is utilized in running Redwood's Equity Skew Strategy.

Disclosures

Definitions and Indices: The S&P 500 Index is a stock market index based on the market capitalizations of 500 leading companies publicly traded in the U.S. stock market, as determined by Standard & Poor's. The Barclays U.S. Corporate High Yield Index is a market value-weighted index which covers the US non-investment grade fixed-rate debt market. UNLESS OTHERWISE NOTED, INDEX RETURNS REFLECT THE REINVESTMENT OF INCOME DIVIDENDS AND CAPITAL GAINS, IF ANY, BUT DO NOT REFLECT FEES, BROKERAGE COMMISSIONS OR OTHER EXPENSES OF INVESTING. INVESTORS MAY NOT MAKE DIRECT INVESTMENTS.

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